MATH PROGRAM CAPSTONE ASSESSMENT

A Summary of Capstone Assessment Activities from Spring 2011 to Spring 2015

1. Introduction

In Spring 2007, the Mathematics Faculty selected both MA411: Introduction to Abstract Algebra and MA422: Introduction to Analysis II as the capstone courses for math majors. Dr. Zoltan Szekely (instructor for MA411) and Dr. Aurora Trance (instructor for MA422) were appointed by the mathematics faculty to administer the Capstone Course Assessments for Spring 2007. In Spring 2011 and Spring 2013, Dr. Szekely administered the Capstone Assessment in MA411. In Spring 2015, the Mathematics Faculty appointed Dr. Henry Taijeron, instructor for MA422, to administer the MA422 Capstone Course Assessment.

The faculty coordinators for the implementation of the Spring 2015 Capstone Course Assessments are the members of the Mathematics Evaluation Committee: Dr. Leslie C. Aquino, Chair for the Committee; Dr. Grazyna Badowski, Chair for the Division of Mathematics and Computer Science; and Dr. Taijeron, instructor for MA422. Dr. Szekely was on sabbatical leave during the academic year 2014-2015 but is now also a member of this Committee. The Committee will continue to oversee development of the Math Program Capstone Assessments during the current assessment cycle.

This report is a summary of capstone assessment activities from previous semesters in which assessments were conducted (in MA411, Spring 2011 and Spring 2013) to the most recent semester in which assessments were conducted (in MA422, Spring 2015). This format used is based on the same format as was done in the Spring 2007 Capstone Course Assessments. (The Spring 2007 report is included as Appendix H, and is a separate attachment.)

In Spring 2007, two direct measures of assessment were used: the Presentation Method by Dr. Szekely for MA411 and the Program Assessment Test by Dr. Trance for MA422. The Presentation Method was used again in Spring 2011 and Spring 2013 for MA411. In Spring 2015, a program assessment test, referred to herein as the Capstone Course Assessment Test (CCAT), was used for the MA422 assessment. The test was approved the fall semester of 2014 by the Division of Mathematics and Computer Science.

It suffices to note that formulation of the capstone course assessment for Spring 2015 was based on the same format as was done in the Spring 2007 Program Assessment Test. The use of rubrics for both Spring 2007 and Spring 2015 were not “officially considered as a means of assessment”. It was in Spring 2015 that rubrics were constructed to report the results and observations to the math faculty. Rubrics were constructed only after the MA422 capstone course assessment was done.

2. The Selected Mathematics Program Learning Outcomes (PLOs)

The Mathematics PLOs were undergoing revision in Spring 2013 and Spring 2015. See Appendix A for both the original and amended PLOs, and Appendix B for course SLOs.

2.1 Mathematics PLOs – Spring 2007

In Spring 2007, the Mathematics Faculty selected **PLO-1** to be assessed.

2.2 Mathematics PLOs – Spring 2011

In Spring 2011, the Mathematics Faculty selected **PLO-3** to be assessed.

2.3 Mathematics PLOs – Spring 2013

In Spring 2013, the Mathematics Faculty selected **PLO-5** to be assessed.

2.4 Mathematics PLOs – Spring 2015

In Spring 2015, the Mathematics Faculty selected **PLO-1**, **PLO-2**, and **PLO-5** to be assessed. Please note that these PLOs are amended PLOs done during the academic year 2014-2015. As a consequence, the math faculty are in the "developing stage" regarding assessments on these PLOs.

3. Capstone Assessment in MA411: Introduction to Abstract Algebra The Presentation Method

3.1 Overview of Method, Content and Administration

In Spring 2011 and Spring 2013, the Presentation Method was used to assess students in MA411. Students were given a choice of six presentation topics selected from group theory, all of which had about the *same level of difficulty*. Each student picked a topic and was assigned a time to give his/her presentation during class. *15-minute presentations* were scheduled at the beginning of classes with additional 5 minutes left for questions. *The instructor asked questions* focusing on some part of the presentation that seemed to be problematic. Peers were also requested to provide feedback. Rubrics were used to evaluate each presentation. The benchmark in both Spring 2011 and Spring 2013 was set as expecting at least 70% of the students scoring at least 70% of the maximum possible score.

In Spring 2011, five different items of measurement to assess the presentation were set up, and in Spring 2013, six different items were set up. The *rubrics* applied for each item of assessment have 4 levels. The lowest *unacceptable level* reflects incorrect statements and/or unintelligible sentences. The next, *basic level*, corresponds to correct statements with some important aspect missing. The *proficient level* means correct statements placed in a context including all important information. Finally, the *advanced level* reflects concise, correct statements together with all necessary information needed for a complete understanding. According to the 4 levels of outcome a point value was assigned from 1 (unacceptable level) to 4 (advanced level) for each item of the assessment. A bonus score point was added for students who scored at the top on all 6 assessment measurement items. Thus the score could range from 1 to 5. The overall score was computed as a weighted average of the scores from the instructor and from the peers.

See Appendices C and D for detailed rubrics and results, and a list of presentation topics.

3.2 The Spring 2011 Assessment

The course MA 411: Introduction to Abstract Algebra is a senior level course in mathematics. Students are expected to already have some experience in presenting mathematical ideas, and after completing the course, should be able to communicate effectively using mathematical language. In Spring 2011, PLO-3 was selected for assessment.

Five different items of measurement, assessing the presentation, were set up. The first item measured *general* skills like a clear statement of the topic. The next two items measure presentation skills with *more detail*, including the correct formulation of statements and using exact mathematical language. The last two items measure the *overall performance* as placing the topic into proper mathematical context and providing a convincing impression.

3.2.1 **Assessing PLO-3:** *argue and reason* using mathematics, *read, create* and *write down* logically correct mathematical proofs, *use exact mathematical language* and *communicate mathematics efficiently* orally, in writing and using information technology tools.

The SLOs for the course indicate some skills that students should be able to communicate using exact mathematical language. By *presenting a mathematical topic* they demonstrate these skills. Special attention was paid to their *reasoning in proofs* and to their *ability to answer questions*. This latter also revealed their overall understanding of the topic they presented.

3.3 The Spring 2013 Assessment

In Spring 2013, PLO-5 was selected for assessment. Six different items of measurement to assess the presentation were set up. The first item measured *general* skills like a clear statement of the topic. The next two items measure presentation skills with *more detail*, including the correct formulation of statements and using exact mathematical language. The fourth item measures the ability of placing the topic into *proper mathematical context*. The fifth item measures the *clarity and correctness* of the answers to the questions. The last item scaled the *overall performance* of providing a convincing impression by the presentation.

3.3.1 **Assessing PLO-5:** *show maturity in mathematical knowledge and thinking* that prepares and encourages students to pursue graduate studies in mathematics or in related fields.

The SLOs for the course develop skills that students use, together with their *maturity in mathematical knowledge and thinking*, to present some challenging algebraic result. By *presenting the chosen mathematical topic* they demonstrate these skills. Special attention was paid to their *reasoning in proofs* and to their *ability to answer questions*. This latter also revealed their overall understanding of the topic.

4. Capstone Assessment in MA422: Introduction to Analysis II

The Program Assessment Test Method

4.1 The Spring 2007 Program Assessment
See Appendix H for Dr. Trance's assessment report (separate attachment).

4.2 The Spring 2015 Capstone Course Assessment Test (CCAT):
The proposed CCAT was drafted using GRE's GR0568 Practice test that was submitted to the Division of Mathematics and Computer Science and was approved during the fall 2014 semester. It is crucial that we have valid and reliable assessment results (as stated in the official WASC-UOG Capstone Course Rubric). It is for these reasons that the GRE practice test was selected in order to guarantee valid and reliable cutoffs regarding the "Low-Medium-High" results for each of the PLO selected for assessment and for the overall result of the assessment test.

The CCAT consisted of fifteen (15) questions. Questions 1, 2, 4, 5, and 9 were used to assess PLO-1; questions 3, 6, 7, 8, 12, and 14 were used to assess PLO-2; and questions 10, 11, 13, and 15 were used to assess PLO-5. (See Appendix D for the questions used for each of these PLOs).

4.2.1 Assessing PLO-1: demonstrate critical thinking, problem solving skills and ability to use mathematical methods by identifying, evaluating, classifying, analyzing, synthesizing data and abstract ideas in various contexts and situations.

Assessing PLO-1 is a practical one which only requires basic algebraic skills and testing the learning outcomes of students in the understanding of the theories and their applications in Differential and Integral Calculus (MA422 SLO-1 was applied for this assessment).

4.2.2 Assessing PLO-2: *exhibit a sound conceptual understanding* of the nature of mathematics, and *demonstrate advanced mathematical skills* in mathematical analysis, modern algebra and other mathematical discipline(s).

Assessing PLO-2 is more demanding since it not only requires basic algebraic skills, but tests learning outcomes of students in "exhibiting a sound conceptual understanding" of not only the elementary theories of differential and integral calculus and their applications, but in other disciplines in mathematics such as elementary matrix analysis and elementary mathematical statistics (MA422 SLO-1 and SLO-3 were applied for this assessment).

4.2.3 Assessing PLO-5: *show maturity in mathematical knowledge and thinking* that prepares and encourages students to pursue graduate studies in mathematics or in related fields.

Assessing PLO-5 is the most difficult part of the Assessment Test since it assesses the learning outcomes of students to see if they are ready to pursue graduate study in mathematics or in a related field of study. It requires not only “exhibiting a sound conceptual understanding” of the theories and applications of mathematics especially in algebra and mathematical analysis, but assessing the students' capability in proving the theories (MA422 SLO-1 and SLO-3 were applied for this assessment).

5. Results and Observations

Below is a Summary of Results Rubric for the assessments conducted by faculty from Spring 2011 to Spring 2015, including the results from the initial assessments in Spring 2007 during the previous assessment cycle.

Summary of Results

Selected PLOs	Semester	Capstone Course	Stage	Direct Measure of Assessment	Results		
					Low	Medium	High
PLO-1	Sp. 07	MA411	Initial	Presentation	X		
PLO-1	Sp. 07	MA422	Initial	PAT[1]	X		
PLO-3	Sp. 11	MA411	Developing	Presentation		X	
PLO-5	Sp. 13	MA411	Developing	Presentation	X		
PLO-1	Sp. 15	MA422	Developing	CCAT[2]	X		
PLO-2	Sp. 15	MA422	Developing	CCAT[2]	X		
PLO-5	Sp. 15	MA422	Developing	CCAT[2]		X	
Class Overall Result	Sp. 15	MA422	Developing	CCAT[2]	X		

[1]PAT (Program Assessment Test); [2] CCAT (Capstone Course Assessment Test)

5.1 MA411 Findings

For the Presentation Method in MA411, there were six students assessed in Spring 2011 and four students assessed in Spring 2013. The benchmark of having at least 70% of students scoring at least 70% was met in Spring 2011 but not in Spring 2013. Below is a table of the results, indicating the number of students achieving low-medium-high performance:

Presentation Method Results (MA411)

PLO	Semester	LOW	MEDIUM	HIGH	AVG Score
PLO-3	Sp. 11	1	4	1	74%
PLO-5	Sp. 13	3	1	0	64%

See Appendix C for a detailed listing of results with examples of low-medium-high performance.

The outcome of the Spring 2011 assessment indicates that students have already acquired a good level of presentation skills, although their use of exact mathematical language still could be improved. All courses should contribute to the further improvement of our students’ exact communication skills.

The outcome of the Spring 2013 assessment indicates that students have not acquired a sufficient level of maturity in mathematical knowledge and thinking as was demonstrated by their overall presentation skills. Students need to improve their presentation skills requiring a maturity in mathematical knowledge and thinking. In particular, they need training in understanding and answering basic questions in junior/senior level mathematics. The Mathematics Program should make efforts and develop special strategies to improve of our students' overall maturity level and communication skills in mathematics.

5.2 MA422 Findings

In Spring 2015, the MA422 Course Syllabus informed students that this course was selected by the mathematics faculty as the capstone course for math majors where a capstone course was clearly defined for the students. At the beginning of the semester, the students were informed that they will be required to take the capstone course assessment test the week prior to the final exam week. For incentive purposes, students were informed that they will be given extra credit on their final exam for taking the assessment test. There were 15 students who took the CCAT in MA422 in Spring 2015. Below is a table indicating the cutoff for the low-medium-high categories of the CCAT:

Cutoffs Scores for CCAT (MA422)

PLO/TEST	GRE (%)	UOG (%)	CUTOFFS (%)		
			LOW	MEDIUM	HIGH
PLO-1	78	**51**	**< 60**	60-85	86-100
PLO-2	61	**28**	**<50**	50-70	71-100
PLO-5	52	**45**	<40	**40-60**	61-100
OVERALL TEST RESULTS	64	**40**	**<50**	50-75	76-100

In Spring 2015, our 15 students who took the CCAT did not do very well on the test. According to the cutoff scores determined by the GRE results, they scored in the low range in the PLO-1 assessment, the PLO-2 assessment, and the overall result of the test. They did score in the medium range in the PLO-5 assessment. We credit this to the fact that these students were just taking MA411, MA422 plus other 400-level mathematics courses (or just recently took some of these courses) that are crucial for pursuing a graduate degree in mathematics.

6. Recommendations

From the Spring 2011 assessment, although the benchmark was met, student performance indicates the need for improvement in use of exact mathematical language. All mathematics courses should contribute to the improvement in exact communication skills, both written and oral.

From Spring 2013, student performance indicates a need to improve mathematics-specific presentation skills, as well as the need for training in integrating knowledge and understanding and answering basic questions. We need to develop strategies to improve the overall maturity level and communication skills in mathematics.

In Spring 2015, our 15 students who took the CCAT did not do very well on the test. They scored poorly on questions that could be answered with knowledge of the material covered in the introductory Calculus sequence, MA203, MA204 and MA205. However, they scored in the medium range for questions associated with PLO-5. Recall that PLO-5 is a learning outcome that assesses if students "*Show maturity in mathematical knowledge and thinking that prepares and encourages students to pursue graduate studies in mathematics or in related fields*."

It is for these reasons why we make the following recommendations:

6.1 Offer a 400-level Capstone Mathematics Course covering:

6.1.1 Review of the Concepts Covered in the CCAT

A review of the concepts covered in the three-semester Calculus sequence: MA203, MA204 and MA205. This should also include a review on linear algebra, the foundations of mathematics, and statistics.

6.1.2 Research Paper in Mathematics

Students should be required to present a mathematical topic with a concentration on mathematical proofs. This will also give students practice in reading and understanding mathematical literature.

6.1.3 The Two Direct Methods Measures

Two direct measures of assessment be implemented at the end of the semester: the Presentation Method and the CCAT.

6.2 Increase the contact hours for the Calculus sequence

6.2.1 Increase contact hours in MA203, MA204 from 5 to 6 contact hours

Additional contact hours in these first two courses in the calculus sequence will better prepare our students by providing them with more hours for "workshop problem solving sessions." These sessions give students practice in applying key mathematical concepts, and help improve retention of this material.

6.2.2 Increase credit hours in MA205 from 3 to 4 credit hours

An increase in credit hours (and thus, contact hours) will help bring the final course in the calculus sequence, MA205: Multivariable Calculus, in line with material covered in similar courses at other universities. Currently, we typically do not have time to reach these topics in a three-credit course, but these are topics that are expected to be known when taking the GRE.

Similar changes should be considered for other required mathematics courses.

The Math Evaluation Committee will work to implement recommendations and close the loop, with collaboration and approval from the faculty in the Division of Mathematics and Computer Science. Changes to the hours for the Calculus sequence were recently approved by the Division in Fall 2015, and Dr. Aquino will push forward with the course revision proposals in the necessary channels as the Math Program Representative to the CNAS AAC. We are considering a phased implementation of a single capstone course, emphasizing the two-course capstone experience in Spring 2016 and Spring 2017, with a single capstone course potentially in place by Spring 2018.

7. Appendices

7.1 Appendix A
Mathematics Program Learning Outcomes (PLOs)

7.2 Appendix B
Student Learning Outcomes (SLOs) for MA411, MA422

7.3 Appendix C
Detailed Capstone Assessment Results for MA411 with Examples

7.4 Appendix D
MA411 Presentation Topics and Detailed Capstone Assessment Rubric

7.5 Appendix E
MA422 Detailed Capstone Assessment Rubric and Results

7.6 Appendix F
Graphs of Results for MA422 Capstone Course Assessment (By Mr. Ryan Flores, Adjunct Prof. of Mathematics)

7.7 Appendix G
Questions Selected for Assessing PLO-1, PLO-2, PLO-5, and the CCAT

7.8 Appendix H
Spring 2007 Assessment Report by Dr. Aurora Trance

Submitted by: **Dr. Leslie J. Aquino, Chair, Mathematics Evaluation Committee**
Prepared by: **Dr. Henry Taijeron, Dr. Zoltan Szekely, and Dr. Aquino**
Date submitted: 20 March 2016

APPENDIX A

MATHEMATICS PROGRAM LEARNING OUTCOMES (PLOs)

Mathematics PLOs – Spring 2007, Spring 2011

PLO-1: Demonstrate critical thinking, problem solving skills and ability to use mathematical methods by identifying, evaluating, and classifying, analyzing, synthesizing, data and abstract ideas in various contexts and situations.
PLO-2: Demonstrate the knowledge of current mathematical applications, computing practices and technology uses in industry, and science and education.
PLO-3: Demonstrate ability to use modern software, abstract thinking, and mathematical practices connected to scientific and industrial problems, and demonstrate these skills that are currently used by technologies in society and education.
PLO-4: Perform skills that enable them to evaluate, propose and convey novel solutions to scientific and business problems, etc.
PLO-5: Demonstrate a sense of exploration that enables students to pursue lifelong learning and currency in their careers in mathematics, statistics, education, high-tech and bi-tech industries.

Mathematics PLOs – Spring 2013, Spring 2015

PLO-1: *Demonstrate critical thinking, problem solving skills* and ability to use mathematical methods by *identifying, evaluating, classifying, analyzing, synthesizing* data and abstract ideas in various contexts and situations.
PLO-2: *Exhibit a sound conceptual understanding* of the nature of mathematics, and *demonstrate advanced mathematical skills* in mathematical analysis, modern algebra and other mathematical discipline(s).
PLO-3: *Argue and reason* using mathematics, *read, create* and *write down* logically correct mathematical proofs, *use exact mathematical language* and *communicate mathematics efficiently* orally, in writing and using information technology tools.
PLO-4: *Apply abstract thinking, mathematical methods, models* and *current practices* in the sciences, including state-of-the-art mathematical software, to solve problems in theoretical mathematics or in a diverse area of mathematical applications.
PLO-5: *Show maturity in mathematical knowledge and thinking* that prepares and encourages students to pursue graduate studies in mathematics or in related fields.
PLO-6: *Demonstrate an appreciation of* and *enthusiasm for* inquiry, learning and creativity in mathematical sciences, a sense of exploration that enables them to *pursue lifelong learning* and *up-to-date professional expertise* in their careers through various areas of jobs, including governmental, business or industrial jobs in mathematics, related sciences, education or technology.

APPENDIX B

STUDENT LEARNING OUTCOMES (SLOs)

<u>Student Learning Outcomes (SLOs)</u>

MA411: Introduction to Abstract Algebra

In this course students will acquire the skill of abstract mathematical thinking through observation, investigation and generalization of features of abstract algebraic structures. In particular, after completing the course, successful students will be able to

- *determine* and *verify* whether a given abstract structure is a group, a ring or neither of the two;
- *recognize* and *apply* the different ways of obtaining new structures from given one like taking subgroups, subrings, subfields, or forming direct sums/products;
- *solve* problems with concrete groups like cyclic groups and permutation groups by applying the intrinsic properties of these groups;
- *compare* algebraic features of mathematical systems through the use of homomorphism and isomorphism;
- *prove* general statements, about properties of groups and rings by using deductive reasoning that proceeds from the defining axioms or from previously established theorems.

MA422: Introduction to Analysis II

<u>SLO1:</u> Demonstrate familiarity with continuity, derivatives, integrals (Riemann), and infinite series (if time permits) of functions.
<u>SLO2:</u> Refine skills in communicating mathematics effectively by participating in classroom discussions and presenting work orally in class.
<u>SLO3:</u> Refine skill in reading, writing, and ascertaining the validity of proofs.

APPENDIX C

DETAILED CAPSTONE ASSESSMENT RESULTS FOR MA411 WITH EXAMPLES

MA411 Capstone Assessment Results
with Examples of Low-Medium-High Performance

	Spring 2011	Spring 2013
MA PLO assessed	MA PLO-3 using exact mathematical language	MA PLO-5 show maturity in mathematical knowledge and thinking
Number of students	6	4
Score: low	1	3
Example	Could not explain basic argument of proof.[1] Could not answer specific question.[2]	Was not able to recall definition, important previous results.[3] Could not understand question.[4]
Score: medium	4	1
Example	Recalled related theorem in proper context.[5] Left out substantial case.[6]	Recalled related definitions and previous results in proper context.[7] Omitted significant argument.[8]
Score: high	1	0
Example	Placed argument into proper context. Used exact language.[9] Showed deep insight.[10] Gave clear answer to the question. [11]	
Benchmark	met	not met
Overall Outcome	acceptable level of presentation skills	no sufficient level of maturity in mathematical knowledge and thinking
Recommendation	1. use of exact mathematical language should be improved, 2. all mathematics courses should contribute to improvement in exact communication skills	1. improve mathematics specific presentation skills, 2. need training in integrating knowledge, understanding and answering basic questions , 3. develop strategies to improve of overall maturity level and communication skills in mathematics
Closing the loop	pending	pending

[1] Student was not able to explain how multiplication works in a factorgroup of a group.
[2] Why do you have elements *h* and *k* in the subgroup such that *hb* = *bk*?
[3] Student could not recall the definition of order of group elements and the theorem about the generator elements of a finite cyclic group.
[4] How could the direct product of two cyclic groups to be not cyclic?
[5] Student properly recalled the theorem on the orders of elements in an external direct product of groups.
[6] In the proof of the Euler-Fermat Theorem student left out the case when the mentioned integers are not relatively prime numbers.
[7] Student correctly recalled Lagrange's Theorem to explain subgroups of a factor group.
[8] Student failed to prove that the exhibited mapping is an automorphism.
[9] Student referred correctly to the properties of set product in Abelian groups.
[10] Student proposed a shortcut to the proof.
[11] Under what circumstances could you use the proposed shortcut?

APPENDIX D

MA411 PRESENTATION TOPICS
AND DETAILED CAPSTONE ASSESSMENT RUBRIC

MA411 Presentation Topics

Spring 2011:

1. The Euler-Fermat Theorem
2. Homomorphism of a dihedral group
3. Finding 4-element subgroups in a given group
4. Subgroups of a factor group
5. Coset product as subgroup
6. Determining the possible orders of a subgroup

Spring 2013:

1. The identity is always a product of even numbers of 2-cycles
2. The automorphism group of the modulo n integers
3. The Euler-Fermat Theorem
4. The Orbit Stabilizer Theorem
5. Subgroups of a factor group
6. Criterion for a direct product of groups to be cyclic

The following rubrics were used for feedback and evaluation in Spring 2011:

MA 411 *Introduction to Abstract Algebra* *Rubrics for Presentation*
Name of presenter: ____________________________ Date:__________
Presentation Topic: ____

Name of evaluator: ____________ ____________	*Advanced:* Concise, correct with all info needed for understanding	*Proficient:* Correct stmt's, all important information included	*Basic:* Correct stmt's, but some important aspect is missing	*Unacceptable:* Incorrect statements, unintelligible sentences
1. The *topic* of the presentation was stated clearly				
2. The *statements* were formulated correctly				
3. The presenter used exact mathematical *language*				
4. *Previous results*, lemmas etc. were mentioned and explained (if any)				
5. The presentation was *convincing* and reflected a good understanding of the topic				
Summary				

Other comments:
____
____

The following rubrics were used for feedback and evaluation in Spring 2013:

MA 411 *Introduction to Abstract Algebra* *Rubrics for Presentation*

Name of presenter: ________________________________ Date:____________

Presentation Topic: ____

Name of evaluator: ____________ ____________	*Advanced:* Concise, correct with all info needed for understanding	*Proficient:* Correct stmt's, all important information included	*Basic:* Correct stmt's, but some important aspect is missing	*Unacceptable:* Incorrect statements, unintelligible sentences
1. The *topic* of the presentation was stated clearly				
2. The *statements* were formulated correctly				
3. The presenter used exact mathematical *language*				
4. *Previous results*, lemmas etc. were mentioned and explained (if any)				
5. The question(s) were answered clearly and correctly				
6. The presentation was *convincing* and reflected a good understanding of the topic				
Summary				

Other comments:

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APPENDIX E

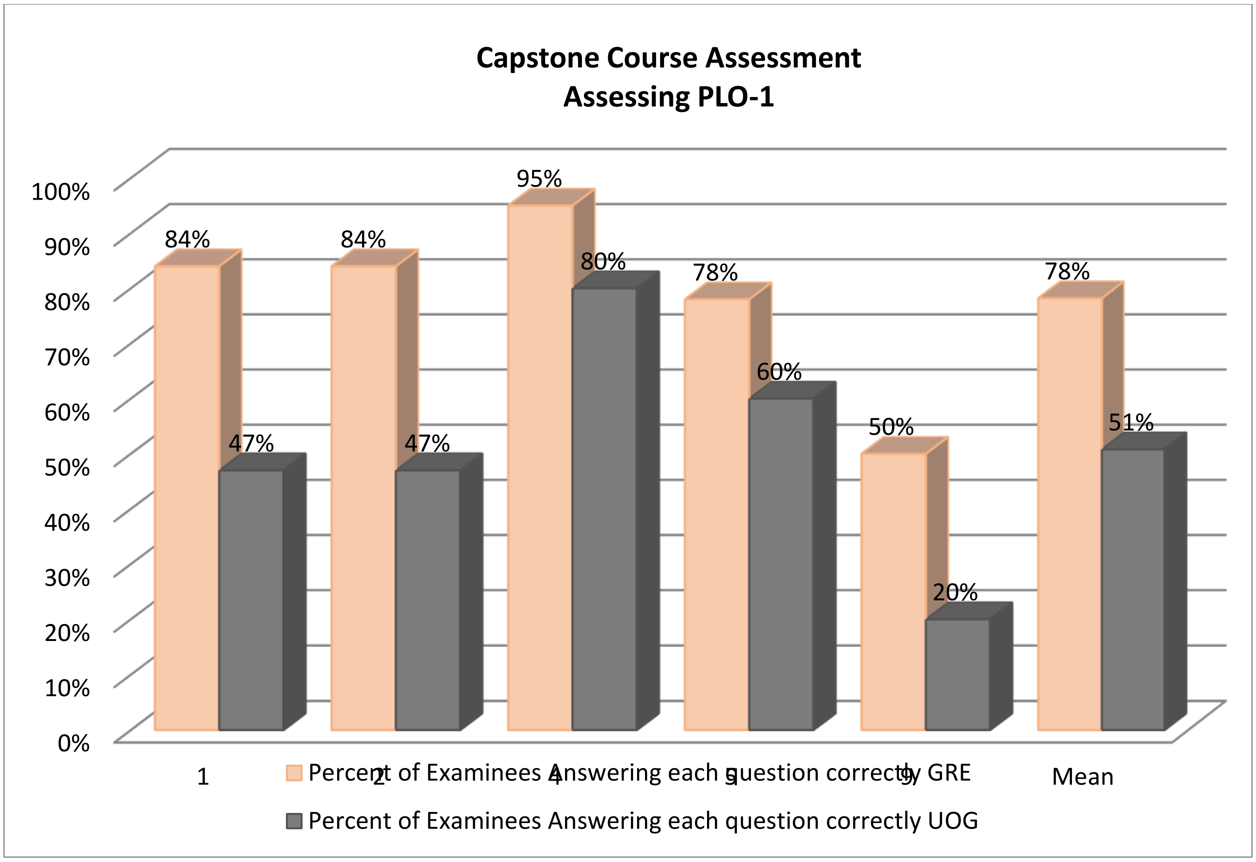
MA422 DETAILED CAPSTONE ASSESSMENT RUBRIC AND RESULTS

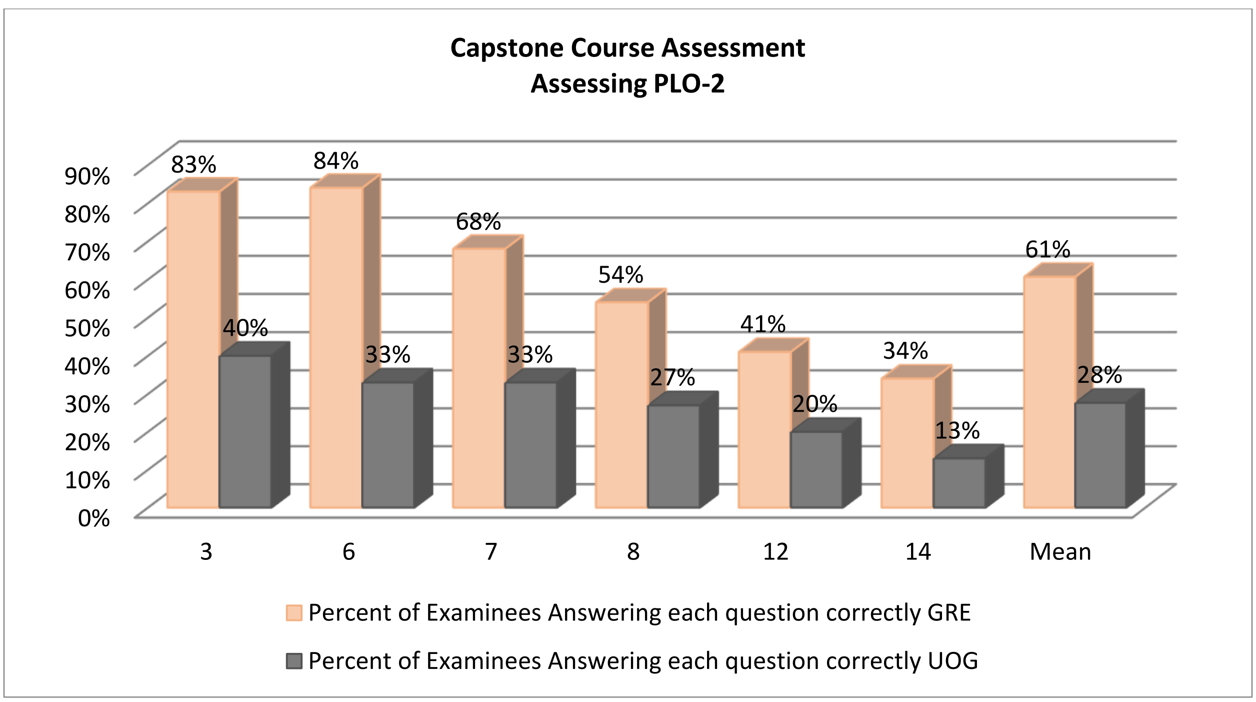
Detailed Math Capstone Course Assessment Rubric (MA422)
Direct Measure of Assessment: Capstone Course Assessment Test (CCAT)
Semester: Spring 2015 Instructor: Dr. Henry Taijeron

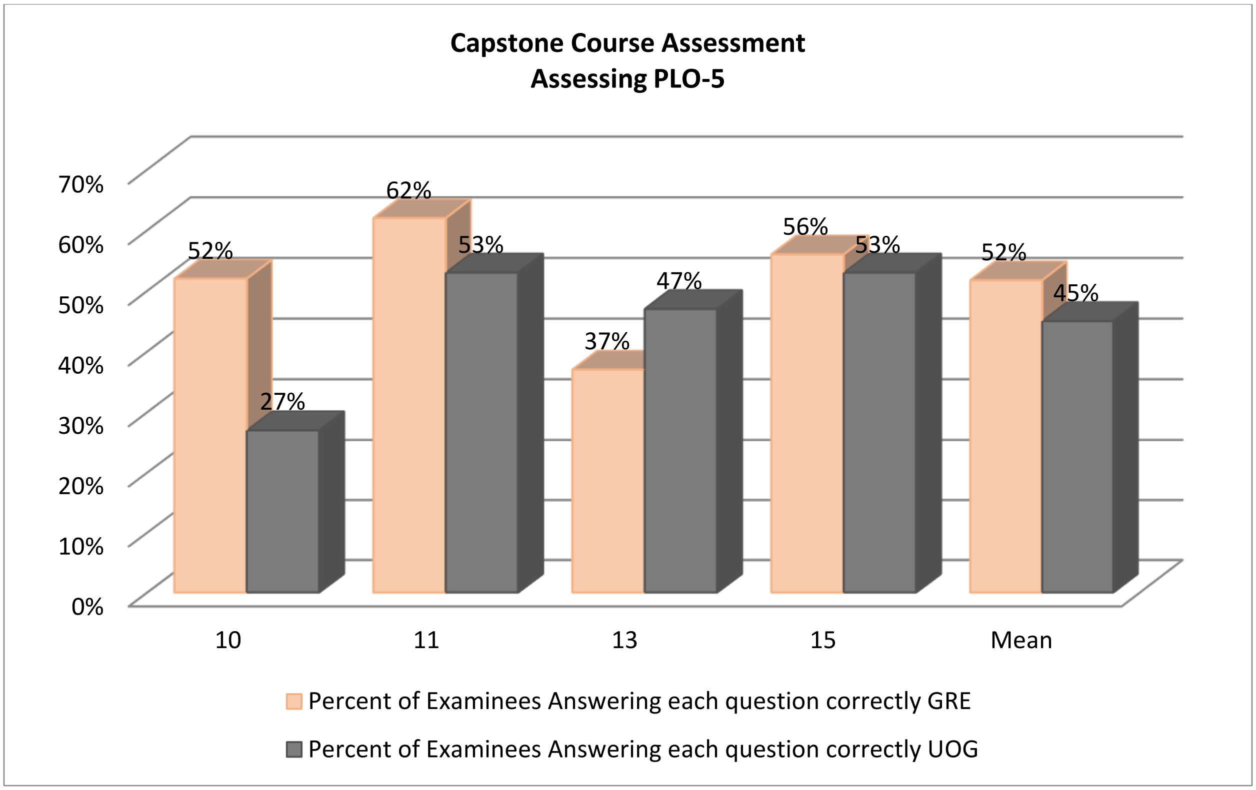
Math PLOs Selected	SLO Applied for the Assessment	CCAT Questions	Results		
			Low	Medium	High
PLO-1: demonstrate critical thinking, problem solving skills and ability to use mathematical methods by identifying, evaluating, classifying, analyzing, synthesizing data and abstract ideas in various contexts and situations.	Assessing the basic algebraic skills and the students' understanding of the theories and their applications in Differential and Integral Calculus (SLO-1).	1, 2, 4, 5, 9	X		
PLO-2: exhibit a sound conceptual understanding of the nature of mathematics, and demonstrate advanced mathematical skills in mathematical analysis, modern algebra and other mathematical discipline(s).	Assessing the students' understanding of the theories and applications of differential and integral calculus, and other disciplines in mathematics such as elementary matrix analysis, elementary mathematical statistics, etc. (SLO-1, SLO-3).	3, 6, 7, 8, 12, 14	X		
PLO-5: show maturity in mathematical knowledge and thinking that prepares and encourages students to pursue graduate studies in mathematics or in related fields.	Assessing the students' understanding to determine if they are ready to pursue graduate study in mathematics or in a related field of study. It requires not only "exhibiting a sound conceptual understanding" of the theories and applications of mathematics especially in algebra and mathematical analysis, but assessing the students' capability in proving the theories (SLO-1, SLO-3).	10, 11, 13, 15		X	
Overall Test Results	Results for all questions on the CCAT	1 - 15	X		

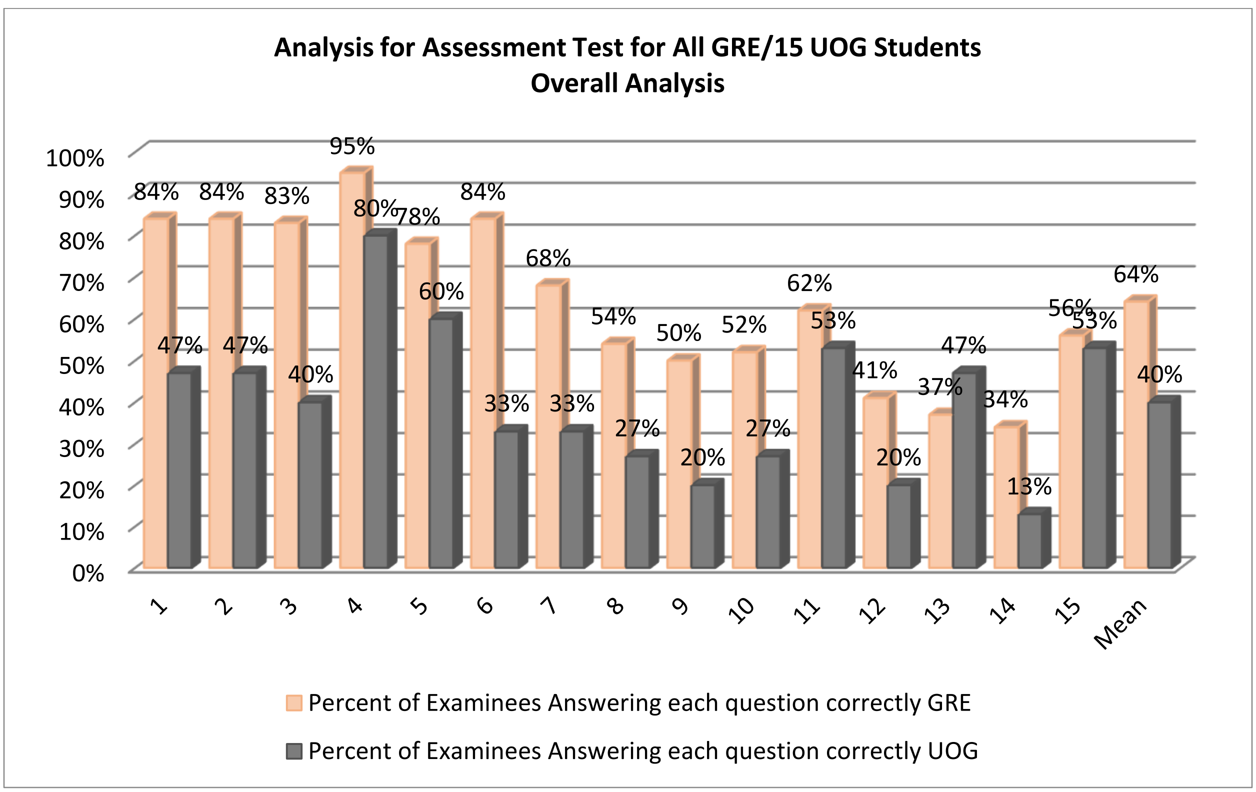
APPENDIX F

GRAPHS OF RESULTS FOR MA422 CAPSTONE ASSESSMENT


Capstone Course Assessment
Assessing PLO-1
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
84%
47%
84%
47%
95%
80%
78%
60%
50%
20%
78%
51%
1
2
4
5
9
Mean
Percent of Examinees Answering each question correctly GRE
Percent of Examinees Answering each question correctly UOG


Capstone Course Assessment
Assessing PLO-2
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
83%
40%
84%
33%
68%
33%
54%
27%
41%
20%
34%
13%
61%
28%
3
6
7
8
12
14
Mean
Percent of Examinees Answering each question correctly GRE
Percent of Examinees Answering each question correctly UOG


Capstone Course Assessment
Assessing PLO-5
70%
60%
50%
40%
30%
20%
10%
0%
52%
27%
62%
53%
37%
47%
56%
53%
52%
45%
10
11
13
15
Mean
Percent of Examinees Answering each question correctly GRE
Percent of Examinees Answering each question correctly UOG


Analysis for Assessment Test for All GRE/15 UOG Students
Overall Analysis
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
84%
47%
84%
47%
83%
40%
95%
80%
78%
60%
84%
33%
68%
33%
54%
27%
50%
20%
52%
27%
62%
53%
41%
20%
37%
47%
34%
13%
56%
53%
64%
40%
1
2
3
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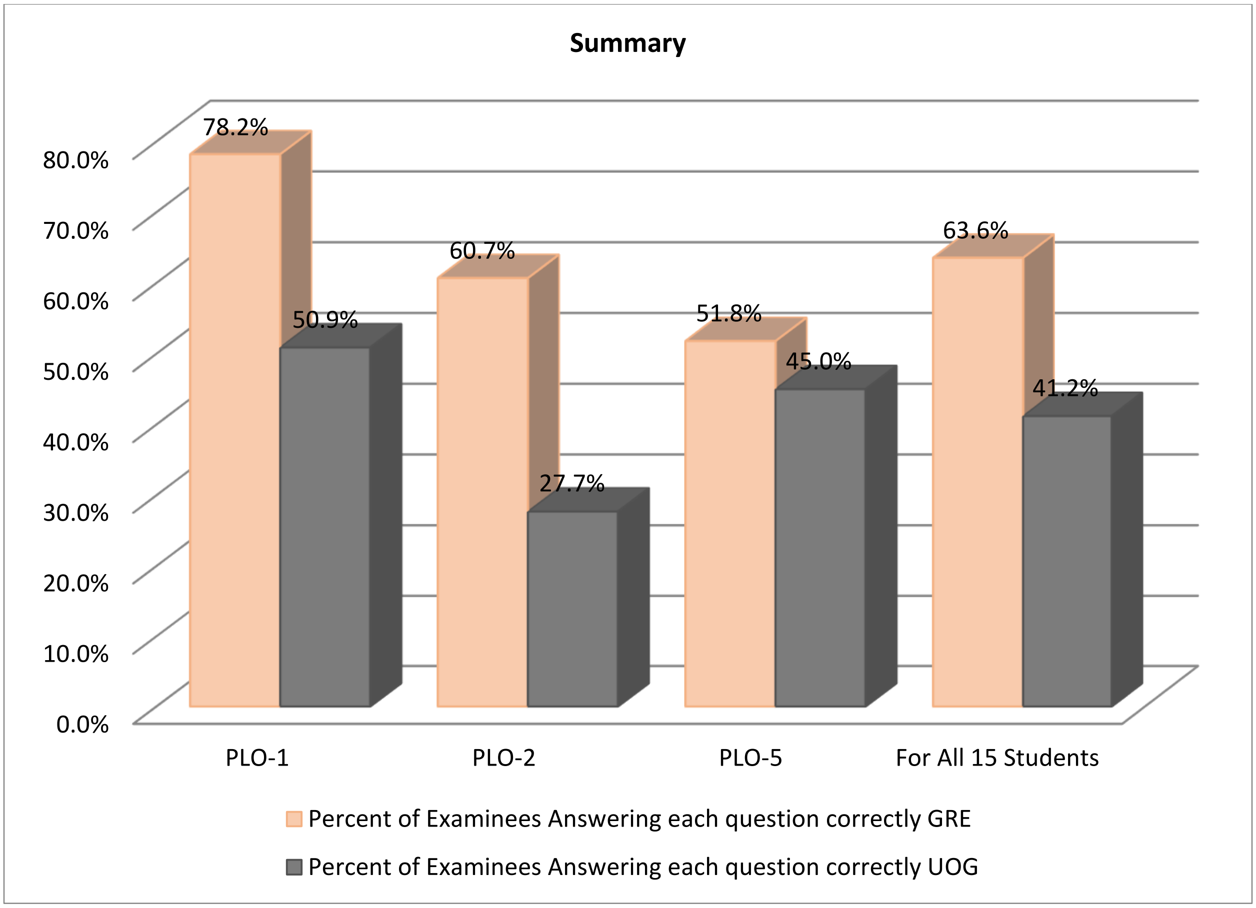
6
7
8
9
10
11
12
13
14
15
Mean
Percent of Examinees Answering each question correctly GRE
Percent of Examinees Answering each question correctly UOG



(Graphs courtesy of Mr. Ryan Flores, Adjunct Professor of Mathematics)

APPENDIX G

QUESTIONS SELECTED FOR CAPSTONE COURSE ASSESSMENT AND THE CCAT

(A) **Assessing PLO-1:** The following questions on the Assessment Test were selected for assessing PLO-1:

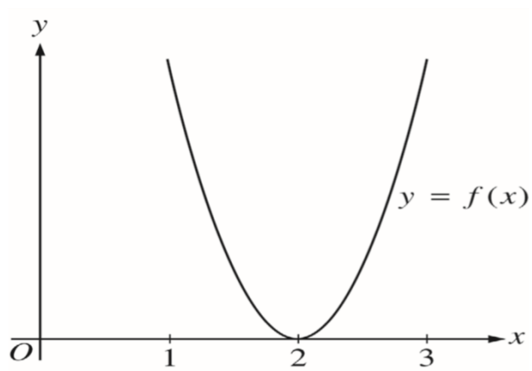
(a) QUESTION 1

In the *xy*-plane, the curve with parametric equations $x = \cos t$ and $y = \sin t$, $0 \le t \le \pi$, has length …

(b) QUESTION 2

Which of the following is an equation of the line tangent to the graph of $y = x + e^x \; at \; x = 0$?

(c) QUESTION 4



Suppose *b* is a real number and $f(x) = 3x^2 + bx + 12$ defines a function on the real line, part of which is graphed above. Then $f(5) =$ …

(d) QUESTION 5

$\int_{-3}^{3} |x+1| dx =$ …

(e) QUESTION 9

Let *h* be the function defined by h(x) = $\int_0^{x^2} e^{x+t} dt$ for all real numbers *x*. Then

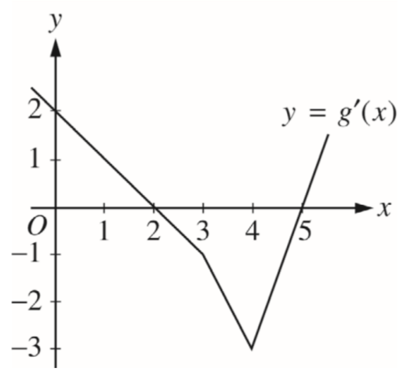
$h'(1) = \ldots$

(B) **Assessing PLO-2:** The following questions on the Assessment Test were selected for assessing PLO-2:

(a) QUESTION 3

If *V* and *W* are 2-dimensional subspaces of $\Re^4$, what are the possible dimensions of the subspace $V \cap W$?

(b) QUESTION 6



Let g be a function whose derivative g' is continuous and has the graph shown above. Which of the following values of g is largest?

(c) QUESTION 7

Let f be a continuous real-valued function defined on the closed interval [-2,3]. Which of the following is NOT necessarily true?

(A) f is bounded.

(B) $\int_{-2}^{3} f(t)dt$ exists.

(C) For each c between f(-2) and f(3), there is an $x \in [-2,3]$ such that f(x) = c.

(D) There is an M in f([-2,3[) such that $\int_{-2}^{3} f(t)dt = 5M$.

(E) $\lim_{h \to 0} it \frac{f(h)-f(0)}{h}$ exists.

(d) QUESTION 8

Let V be the real vector space of all real 2x3 matrices, and let W be the real vector space of all real 4x1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{v \in V \mid T(v) = 0\}$?

(e) QUESTION 12

A fair coin is to be tossed 100 times, with each toss resulting in a head or a tail. If H is the total number of heads and T is the total number of tails, which of the following events has the greatest probability?

(A) $H = 50$

(B) $T \geq 60$

(C) $51 \leq H \leq 55$

(D) $H \geq 48 \; and \; T \geq 48$

(E) $H \leq 5 \; or \; H \geq 95$

(f) QUESTION 14

Let A be a real 2x2 matrix. Which of the following statements must be true?

I. All of the entries of A^2 are nonnegative.

II. The determinant of A^2 is nonnegative.

III. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

(C) **Assessing PLO-5:** The following questions on the Assessment Test were selected for assessing PLO-5:

(a) QUESTION 10

For all positive functions *f* and *g* of the real variable *x*, let ~ be a relation defined by

$$f \sim g \text{ if and only if } \lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$$

Which of the following is NOT a consequence of f ~ g?

(A) $f^2 \sim g^2$

(B) $\sqrt{f} \sim \sqrt{g}$

(C) $e^f \sim e^g$

(D) $f + g \sim 2g$

(E) $g \sim f$

(b) QUESTION 11

Let f be a function from a set X to a set Y. Consider the following statements:

P: For each $x \in X$, there exists $y \in Y$ such that $f(x) = y$.

Q: For each $y \in Y$, there exists $x \in X$ such that $f(x) = y$.

R: There exist $x_1, x_2 \in X$, such that $x_1 \neq x_2$ *and* $f(x_1) = f(x_2)$.

The negation of the statement “f is one-to-one and onto Y” is

(A) P or not R

(B) R or not P

(C) R or not Q

(D) P and not R

(E) R and not Q

(c) QUESTION 13

Consider the theorem: If f *and* f' are both strictly increasing real-valued functions on the interval $(0,\infty)$, then $\lim_{x\to\infty} f(x) = \infty$. Then the following argument is suggested as a proof of this theorem:

(a) By the Mean Value Theorem, there is a c_1 in the interval (1,2) such that
$$f'(c_1) = \frac{f(2)-f(1)}{2-1} = f(2)-f(1) > 0.$$

(b) For each $x > 2$, there is a c_x *in* $(2,x)$ *such that* $\frac{f(x)-f(2)}{x-2} = f'(c_x)$.

(c) For each $x > 2$, $\frac{f(x)-f(2)}{x-2} = f'(c_x) > f'(c_1)$ since f' is strictly increasing.

(d) For each $x > 2$, $f(x) > f(2) + (x-2)f'(c_1)$.

(e) $\lim_{x\to\infty} f(x) = \infty$.

Which of the following statements is true?

(A) The argument is valid.

(B) The argument is not valid since the hypothesis of the Mean Value Theorem are not satisfied in (a) and (b).

(C) The argument is not valid since (c) is not valid.

(D) The argument is not valid since (d) cannot be deduced from the previous steps.

(E) The argument is not valid since (d) does not imply (e).

(d) QUESTION 15

Suppose that two binary operations, denoted by $\oplus$ *and* $\otimes$, are defined on a nonempty set S, and that the following conditions are satisfied for all x, y, and z in S:

(a) $x \oplus y$ *and* $x \otimes y$ are in S.

(b) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ *and* $x \otimes (y \otimes z) = (x \otimes y) \otimes z$.

(c) $x \oplus y = y \oplus x$

Also, for each *x* in *S* and for each positive integer *n*, the elements *nx* and x^n are defined recursively as follows:

$1x = x^1 = x$ and

If kx *and* x^k have been defined, then $(k+1)x = kx \oplus x$ *and* $x^{k+1} = x^k \otimes x$.

Which of the following must be true?

I. $(x \otimes y)^n = x^n \otimes y^n$ for all x and y in S and for each positive integer n.

II. $n(x \oplus y) = nx \oplus ny$ for all x and y in S and for each positive integer n.

III. $x^m \otimes x^n = x^{m+n}$ for each x in S and for all positive integers m and n.

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

MA422
THE CAPSTONE COURSE ASSESSMENT TEST (CCAT)
(COURTESY OF GRE – FORM GR0568)
SPRING 2015

NOTE: In this test:

1. All logarithms with an unspecified base are natural logarithms, that is, with base e.
2. The set of all real numbers x such that $a \le x \le b$ is denoted by [a,b].
3. The symbols $Z, Q, \Re,$ *and* ζ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

(1) In the *xy*-plane, the curve with parametric equations $x = \cos t$ and $y = \sin t$, $0 \le t \le \pi,$ has length

(A) 3
(B) π
(C) 3π
(D) $\frac{3}{2}$
(E) $\frac{\pi}{2}$

(2) Which of the following is an equation of the line tangent to the graph of $y = x + e^x$ *at* $x = 0$?

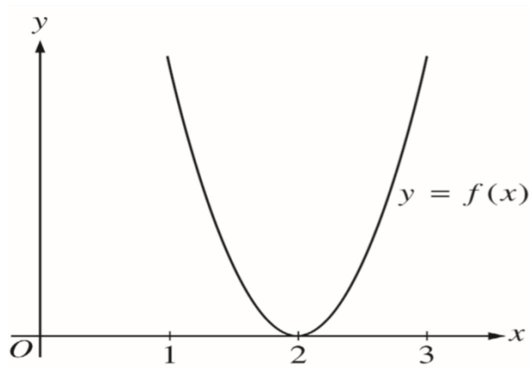
a. $y = x$
b. $y = x+1$
c. $y = x+2$
d. $y = 2x$
e. $y = 2x+1$

(3) If V and W are 2-dimensional subspaces of $\Re^4$, what are the possible dimensions of the subspace $V \cap W$?

a. 1 only
b. 2 only

c. 0 and 1 only
d. 0, 1, and 2 only
e. 0, 1, 2, 3, and 4

(4)



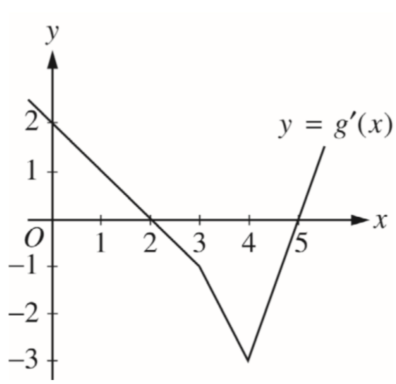
Suppose b is a real number and $f(x) = 3x^2 + bx + 12$ defines a function on the real line, part of which is graphed above. Then $f(5) =$

(A) 15
(B) 27
(C) 67
(D) 72
(E) 87

(5) $\int_{-3}^{3} |x+1| dx =$

(A) 0
(B) 5
(C) 10
(D) 15
(E) 20

(6)



Let g be a function whose derivative g' is continuous and has the graph shown above. Which of the following values of g is largest?

(A) g(1)
(B) g(2)
(C) g(3)
(D) g(4)
(E) g(5)

(7) Let f be a continuous real-valued function defined on the closed interval [-2,3]. Which of the following is NOT necessarily true?

(F) f is bounded.
(G) $\int_{-2}^{3} f(t)dt$ exists.
(H) For each c between f(-2) and f(3), there is an $x \in [-2,3]$ such that f(x) = c.
(I) There is an M in f([-2,3[) such that $\int_{-2}^{3} f(t)dt = 5M$.
(J) $\lim_{h\to 0} it\ \frac{f(h)-f(0)}{h}$ exists.

(8) Let V be the real vector space of all real 2x3 matrices, and let W be the real vector space of all real 4x1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{v \in V \mid T(v) = 0\}$?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

(9) Let h be the function defined by h(x) = $\int_0^{x^2} e^{x+t}dt$ for all real numbers x. Then $h'(1) =$

(A) e – 1
(B) e^2
(C) $e^2 - e$
(D) $2e^2$
(E) $3e^2 - e$

(10) For all positive functions f and g of the real variable x, let ~ be a relation defined by

$$f \sim g \text{ if and only if } \lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$$

Which of the following is NOT a consequence of f ~ g?

(F) $f^2 \sim g^2$

(G) $\sqrt{f} \sim \sqrt{g}$

(H) $e^f \sim e^g$

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The negation of the statement "f is one-to-one and onto Y" is

(F) P or not R

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(H) R or not Q

(I) P and not R

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(12) A fair coin is to be tossed 100 times, with each toss resulting in a head or a tail. If H is the total number of heads and T is the total number of tails, which of the following events has
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(g) For each $x > 2$, there is a c_x *in* $(2, x)$ *such that* $\frac{f(x) - f(2)}{x-2} = f'(c_x)$.

(h) For each $x > 2$, $\frac{f(x) - f(2)}{x-2} = f'(c_x) > f'(c_1)$ since f' is strictly increasing.

(i) For each $x > 2$, $f(x) > f(2) + (x-2) f'(c_1)$.

(j) $\lim_{x \to \infty} it\ f(x) = \infty$.

Which of the following statements is true?

(F) The argument is valid.

(G) The argument is not valid since the hypothesis of the Mean Value Theorem are not satisfied in (a) and (b).

(H) The argument is not valid since (c) is not valid.

(I) The argument is not valid since (d) cannot be deduced from the previous steps.

(J) The argument is not valid since (d) does not imply (e).

(14) Let A be a real 2x2 matrix. Which of the following statements must be true?

IV. All of the entries of A^2 are nonnegative.

V. The determinant of A^2 is nonnegative.

VI. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(F) I only

(G) II only

(H) III only

(I) II and III only

(J) I, II, and III

(15) Suppose that two binary operations, denoted by $\oplus$ *and* $\otimes$, are defined on a nonempty set S, and that the following conditions are satisfied for all x, y, and z in S:

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VI. $x^m \otimes x^n = x^{m+n}$ for each x in S and for all positive integers m and n.

(F) I only

(G) II only

(H) III only

(I) II and III only

(J) I, II, and III

APPENDIX H

SPRING 2007 PROGRAM ASSESSMENT REPORT

(See Attached File)